GLOBAL STABILITY OF A FRACTIONAL PARTIAL DIFFERENTIAL EQUATION

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Dedicated to Professor S.-O. Londen on the occasion of his 60th birthday.

1. Introduction. We consider the equation

\begin{equation}
\frac{\partial u}{\partial t}(t, x) = \int_0^t b(t - s) \frac{\partial^2 u}{\partial x^2}(s, x) ds + (g(u_x(t, x)))_x, \\
t > 0, \quad x \in (0, 1)
\end{equation}

with boundary conditions

\begin{equation}
\begin{aligned}
&u(t, 0) = u(t, 1) = 0, \quad t > 0, \\
\end{aligned}
\end{equation}

and initial values

\begin{equation}
\begin{aligned}
&u(0, x) = u^0(x), \quad u_t(0, x) = u^1(x).
\end{aligned}
\end{equation}

This problem is motivated by the theory of viscoelastic materials, cf., Pego [10], Ball et al. [1]. The nonlinearity \( g \) behaves typically like a power at infinity,

\[
g(\xi) \sim \text{sign}(\xi)|\xi|^m, \quad |\xi| \to \infty,
\]

and we assume that it vanishes solely at zero so that zero is the unique stationary state of (1.1).

The convolution term represents a fractional derivative with respect to \( t \); explicitly, let

\[
b(t) = \frac{t^{-\alpha}}{\Gamma(1 - \alpha)}, \quad 0 < \alpha < 1.
\]