TIME ASYMPTOTIC BEHAVIOR OF THE
SOLUTION TO A CAUCHY PROBLEM
GOVERNED BY A TRANSPORT OPERATOR

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ABSTRACT. The purpose of this paper is to investigate
some spectral properties of a time-dependent linear transport
equation with diffusive boundary conditions arising in growing
cell populations. After deriving the explicit expression of the
strongly continuous semigroup \{U^K(t) : t \geq 0\} generated by
the streaming operator, we establish the strict singularity of
the operator \(BU^K(s)B\), \(s > 0\), for a wide class of collision
operators \(B\). Making use of the Weis theorem, Theorem 4.1,
this enables us to estimate the essential type of the transport
semigroup from which the asymptotic behavior of the solution
is derived.

1. Introduction. In this paper we deal with the well-posedness and
the time asymptotic behavior of solutions to transport equations for a
sizable class of scattering operators. More precisely, we are concerned
with the following initial boundary value problem

\[
\begin{aligned}
\frac{\partial \psi}{\partial t}(\mu, v, t) &= -v\frac{\partial \psi}{\partial \mu}(\mu, v, t) - \sigma(v)\psi(\mu, v, t) \\
&\quad + \int_0^c r(\mu, v, v')\psi(\mu, v', t) \, dv' \\
A_K\psi(\mu, v, t) &= S_K\psi(\mu, v, t) + B\psi(\mu, v, t), \\
\psi(\mu, v, 0) &= \psi_0(\mu, v),
\end{aligned}
\]

where \(\mu \in [0, a]\), \(v, v' \in [0, c]\) with \(a > 0\) and \(c > 0\), \(S_K\) denotes
the streaming operator and \(B\) stands for the collision one (the integral
part of \(A_K\)). This model describes the number density \(\psi(\mu, v, t)\) of cell
population as a function of the degree of maturity \(\mu \in [0, a]\), \(a > 0\),
the maturation velocity \(v \in [0, c]\), \(c > 0\), and the time \(t\). The degree
of maturation is defined so that \(\mu = 0\) at the birth and \(\mu = c\) at
mitosis, i.e. cells born at \(\mu = 0\) and divided at \(\mu = c\). The kernel
\(r(\mu, v, v')\) is the transition rate. It specifies the transition of cells from
the maturation velocity \(v'\) to \(v\) while \(\sigma(v)\) denotes the total transition.