THE SPECTRAL THEORY OF SECOND ORDER
TWO-POINT DIFFERENTIAL OPERATORS
IV. THE ASSOCIATED PROJECTIONS
AND THE SUBSPACE $S_\infty(L)$

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ABSTRACT. This paper is the final part in a four-part
series on the spectral theory of a two-point differential oper-
ator $L$ in $L^2[0,1]$, where $L$ is determined by a formal differ-
ential operator $l = -D^2 + q$ and by independent boundary
values $B_1, B_2$. For the family of projections \( \{Q_k\}_{k=1}^{\infty} \cup \{Q'_k\}_{k=0}^{\infty} \) which map $L^2[0,1]$ onto the general-
ized eigenspaces of $L$, it is determined whether or not the
family of all finite sums of these projections is uniformly
bounded in norm. Equivalently, for the subspace $S_\infty(L)$
consisting of all $u \in L^2[0,1]$ with $u = \sum_{k=1}^{\infty} Q_0 u +
\sum_{k=0}^{\infty} Q'_k u$, it is determined whether or not
$S_\infty(L) = \overline{S_\infty(L)} = L^2[0,1]$. It is necessary to modify the
projections and $S_\infty(L)$ in the multiple eigenvalue case.

1. Introduction. In this paper we conclude our four-part series
on the spectral theory of a linear second order two-point differential
operator $L$ in the complex Hilbert space $L^2[0,1]$. Let $L$ be the
differential operator in $L^2[0,1]$ defined by

$$D(L) = \{u \in H^2[0,1] \mid B_i(u) = 0, \ i = 1, 2\},$$

$$Lu = l u,$$

where

$$l = -\left( \frac{d}{dt} \right)^2 + q(t) \left( \frac{d}{dt} \right)^0$$

is a second order formal differential operator on the interval $[0,1]$ with
$q \in C[0,1]$, $B_1, B_2$ are linearly independent boundary values given by

$$B_1(u) = a_1 u'(0) + b_1 u'(1) + a_0 u(0) + b_0 u(1),$$

$$B_2(u) = c_1 u'(0) + d_1 u'(1) + c_0 u(0) + d_0 u(1),$$

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