COINCIDENCE PRINCIPLES AND FIXED POINT THEORY FOR MAPPINGS IN LOCALLY CONVEX SPACES

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ABSTRACT. We present a coincidence principle for concentrative maps. This leads to new fixed point theory for nonlinear operators.

1. Introduction. A general coincidence theory is presented for concentrative mappings between locally convex Hausdorff linear topological spaces in this paper. These general results are used to obtain a variety of new fixed point theorems for the sum of two operators, for example, an $m$-accretive plus a condensing operator, between Banach spaces (one could also obtain results for operators between locally convex Hausdorff linear topological spaces). The fixed point results were motivated from a variety of sources, in particular we mention the work of Browder [4], Daneš [7], Furi and Pera [14], Gatica and Kirk [15], Granas [16], Petryshyn [25], Precup [26], Reinermann [27] and Schöneberg [28]. Some applications of our results are also presented in this paper.

For the remainder of this section we gather together some definitions and some known facts. Let $(E, d)$ be a pseudometric space [18] and $M$ a subset of $E$. For $x \in M$, let $B(x, \varepsilon)$ denote the closed $\varepsilon$-ball with center $x$, i.e., $B(x, \varepsilon) = \{ y \in E : d(x, y) \leq \varepsilon \}$. The measure of noncompactness of the set $M$ is defined by

$$
\alpha(M) = \inf Q(M); \quad \inf(\emptyset) = \infty,
$$

where

$$
Q(M) = \{ \varepsilon \in \mathbb{R} : \varepsilon > 0 \text{ and there is a finite } \varepsilon\text{-net for } M \text{ in } E \}
$$

i.e., $M \subseteq B(A, \varepsilon)$ for some finite subset $A$ of $E$.

Note $B(A, \varepsilon) = \{ x \in E : \inf \{ d(x, y) : y \in A \} \leq \varepsilon \}$. 

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