MATRIX SUMMABILITY METHODS AND WEAKLY UNCONDITIONALLY CAUCHY SERIES

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ABSTRACT. We study new sequence spaces determined by series in normed spaces and a matrix summability method, giving new characterizations of weakly unconditionally Cauchy series. We obtain characterizations for the completeness of a normed space, and a version of the Orlicz-Pettis theorem via matrix summability methods is also proved.

1. Introduction. Let $X$ be a real normed space. A series $\sum x_i$ in $X$ is said to be unconditionally convergent (uc) if $\sum x_{\pi(i)}$ converges for every permutation $\pi$ of $\mathbb{N}$. We say that $\sum x_i$ is weakly unconditionally Cauchy (wuc) if, for every permutation $\pi$ of $\mathbb{N}$, we have that the sequence $(\sum_{i=1}^n x_{\pi(i)})_n$ is weakly Cauchy. It is a well-known fact, see [5], that $\sum x_i$ is wuc if and only if $\sum |f(x_i)| < \infty$ for every $f \in X^*$, where $X^*$ denotes the dual space of $X$. The following results are also well known, see [3, 5, 6]:

Let $X$ be a Banach space, and let $\sum x_i$ be a series in $X$. Then:
1. $\sum x_i$ is uc if and only if $\sum a_i x_i$ is convergent for every $(a_i)_i \in l_\infty$.
2. $\sum x_i$ is wuc if and only if $\sum a_i x_i$ is convergent for every $(a_i)_i \in c_0$.
3. There exists a series $\sum x_i$ wuc and not uc in $X$ if and only if $X$ has a copy of $c_0$.

The following concepts and definitions can be found in [4].

A matrix method of limit is defined by a matrix $A = (a_{ij})_{(i,j) \in \mathbb{N} \times \mathbb{N}}$ of real entries in the following way: If $(x_i)_i$ is a sequence in a normed space $X$, we say that $A \lim_i x_i = x_0$ if, for every $i \in \mathbb{N}$, the series $\sum_j a_{ij} x_j$ is convergent and $\lim_i \sum_j a_{ij} x_j = x_0$. 


Keywords and phrases. Matrix summability, weakly unconditionally Cauchy series, Orlicz-Pettis theorem.

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